Name: ....

Every exercise is for 5 point, thus 40 points in total.

1. Use the Cholesky factorization of the real matrix

$$\mathbf{A} = \begin{pmatrix} 9 & 6 & 3 & 3 & 0 \\ 6 & 5 & 1 & 2 & 1 \\ 3 & 1 & 6 & 3 & 1 \\ 3 & 2 & 3 & 11 & -2 \\ 0 & 1 & 1 & -2 & 4 \end{pmatrix}$$

to solve the system $\mathbf{Ax} = (48, 35, 13, 40, -4)^T$

2. Use the projection to find the best approximate solution $\mathbf{x}'$ of the system $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 3 & 2 & -1 & 1 & 3 \\ 3 & 1 & 3 & -1 & -2 \\ 2 & -3 & -1 & -1 & -1 \\ 1 & -1 & -2 & -3 & 1 \\ 1 & -3 & 1 & 3 & 1 \\ 1 & 1 & -3 & 2 & -3 \end{pmatrix}, \qquad \mathbf{b} = (26, 5, 34, -18, -30, -13)^T$$

Determine the error $||\mathbf{Ax}' - \mathbf{b}||$.

Observe that the columns of $\mathbf{A}$ are mutually perpendicular.

3. Determine the basis of the orthogonal complement of the row space of the following matrix. Also do the same task for its column space. (Both w.r.t. the standard scalar product.)

$$\begin{pmatrix} 2 & -4 & 2 & 0 & 8 \\ 3 & -6 & 2 & 3 & 13 \\ 5 & -10 & 1 & 12 & 24 \\ 2 & -4 & -1 & 9 & 9 \end{pmatrix}$$

4. In the space $\mathbb{R}^4$ find the orthonormal basis of the row space of the following matrix

$$\begin{pmatrix} -4 & 1 & -2 & 2 \\ -6 & 4 & -8 & 3 \\ 8 & 3 & -6 & -4 \\ 12 & 3 & 14 & -1 \end{pmatrix}$$

5. Determine eigenvalues of the matrix

$$\begin{pmatrix} 3 & 1 & 0 & 0 & -2 \\ 0 & 5 & 0 & 0 & 0 \\ -2 & 3 & 4 & 0 & 4 \\ 0 & 5 & 6 & 2 & 1 \\ 4 & 8 & 0 & 0 & -3 \end{pmatrix}.$$

6. Transform the following complex matrix into Jordan normal form and determine eigenvectors, and if necessary also generalized eigenvectors.

$$\begin{pmatrix} 4 & -3 & 1 & 0 \\ 6 & -5 & 2 & 0 \\ 4 & -4 & 2 & 0 \\ -1 & 1 & -1 & 1 \end{pmatrix}$$

7. Calculate

a)
$$
\left|\begin{array}{ccccccccccc}
10 & 10 & 10 & \cdots & & & & \cdots & 10 & 10 & 10 \\
0 & 9 & 9 & \cdots & & & & \cdots & 9 & 9 & 0 \\
0 & 0 & 8 & \cdots & & & & \cdots & 8 & 0 & 0 \\
\vdots & & \ddots & \ddots & & & & \iddots & \iddots & & \vdots \\
 & & & 0 & 2 & 2 & 2 & 0 & & & \\
 & & & 0 & 0 & 1 & 0 & 0 & & & \\
 & & & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & & & \\
\vdots & & \iddots & \iddots & & & & \ddots & \ddots & & \vdots \\
0 & 0 & -\frac{1}{8} & \cdots & & & & \cdots & \frac{1}{8} & 0 & 0 \\
0 & -\frac{1}{9} & \frac{1}{9} & \cdots & & & & \cdots & \frac{1}{9} & \frac{1}{9} & 0 \\
-\frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \cdots & & & & \cdots & \frac{1}{10} & \frac{1}{10} & \frac{1}{10}
\end{array}\right|
$$

b)
$$
\left|\begin{array}{ccccccccccccccccc}
n & 0 & 0 & \cdots & & & & & & & & & & \cdots & 0 & 0 & n \\
n & n-1 & n-1 & n-1 & \cdots & & & & & & & & \cdots & n-1 & n-1 & n-1 & n \\
n & 0 & n-2 & 0 & 0 & \cdots & & & & & & \cdots & 0 & 0 & n-2 & 0 & n \\
\vdots & 0 & n-2 & n-3 & n-3 & \cdots & & & & & & \cdots & n-3 & n-3 & n-2 & 0 & \vdots \\
 & \vdots & \vdots & & & & & & & & & & & & \vdots & \vdots & \\
 & & & & & 4 & 0 & 0 & 0 & 0 & 0 & 4 & & & & & \\
 & & & & & 4 & 3 & 3 & 3 & 3 & 3 & 4 & & & & & \\
 & & & & & 4 & 0 & 2 & 0 & 2 & 0 & 4 & & & & & \\
 & & & & & 4 & 0 & 2 & 1 & 2 & 0 & 4 & & & & & \\
 & & & & & 4 & 0 & 2 & 0 & -2 & 0 & 4 & & & & & \\
 & & & & & 4 & 3 & 3 & 3 & 3 & -3 & 4 & & & & & \\
 & & & & & 4 & 0 & 0 & 0 & 0 & 0 & -4 & & & & & \\
 & \vdots & \vdots & & & & & & & & & & & & \vdots & \vdots & \\
\vdots & 0 & n-2 & n-3 & n-3 & \cdots & & & & & & \cdots & n-3 & 3-n & n-2 & 0 & \vdots \\
n & 0 & n-2 & 0 & 0 & \cdots & & & & & & \cdots & 0 & 0 & 2-n & 0 & n \\
n & n-1 & n-1 & n-1 & \cdots & & & & & & & & \cdots & n-1 & n-1 & 1-n & n \\
n & 0 & 0 & \cdots & & & & & & & & & & \cdots & 0 & 0 & -n
\end{array}\right|
$$