Next time - test!

Exercise 1: Find an orthogonal complement of a space in $\mathbb{R}^3$ generated by vectors $\mathbf{u} = (1,2,3)^T, \mathbf{v} = (1,-1,0)^T$ with respect to the standard inner product.

Exercise 2: Find a projection of a vector $\mathbf{x} = (1,2,3,4,5)^T$ to a vector space V generated by vectors $\mathbf{x}_1 = (1,0,1,0)^T, \mathbf{x}_2 = (1,1,1,1)^T, \mathbf{x}_3 = (1,0,0,1)^T$. Find the distance of x from V with respect to the standard inner product.

Exercise 3: The population data are the following:

year	1950	1960	1970	1980	1990	2000
population	2519	2982	3692	4435	5263	6070

Find a linear dependency of the population on time and estimate the population in 2009.

Exercise 4: Suppose A is an $n \times k$ matrix, where $k \leq n$, such that the columns of A are linearly independent. Then the $k \times k$ matrix $A^T A$ is invertible.

Exercise 5: Suppose that M is an $n \times n$ matrix such that $M^T = M = M^2$. Let W denote the column space of M.

1. Suppose that $Y \in W$. Prove that $MY = Y$.
2. Suppose that v is a vector in $\mathbb{R}^n$. Why is $Mv \in W$?
3. If $Y \in W$, why is $v - Mv \perp Y$?
4. Conclude that Mv is the projection of v into W.

Exercise 6: Use the projection to find the best approximate solution of the system $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A} = \begin{pmatrix} 2 & 1 & 0 \\ 4 & 2 & 0 \\ 2 & -4 & -1 \\ 1 & -2 & 2 \end{pmatrix}, \qquad \mathbf{b} = (10,5,13,9)^T.$

Observe that the columns of $\mathbf{A}$ are mutually perpendicular.

Exercise 7: Prove (again?) that projection is a linear map.