NMAG403 - Combinatorics

November 24, 2023 – Planarity and Hamiltonicity

Homework

Deadline: **December 18, 2023**
Send to: `honza@kam.mff.cuni.cz` (in PDF)

1. Let Q_n be the n-dimensional hypercube graph, i.e. the vertices of Q_n are all $\{0,1\}$-strings of length n and two strings are connected by an edge if and only if they differ in exactly one position. Prove that Q_n has a Hamiltonian cycle, provided $n \geq 2$.
2. Prove that in a bipartite graph whose all vertices of one class of bipartition have odd degrees, the total number of Hamiltonian cycles is even.
3. Show that if $c_v(G) \geq \alpha(G)$ holds true for a graph G with at least 3 vertices, then G has a Hamiltonian cycle.
4. * Construct a vertex-3-connected graph with exactly one Hamiltonian cycle. (An original solution of this problem will gain you the "zápočet".)
5. ** Does there exist a planar vertex-3-connected graph with exactly one Hamiltonian cycle? (A solution of this problem will gain you an A-grade from this course.)

In class problems

36. **(Fáry embedding)** Prove that every planar graph has a planar non-crossing drawing in which all edges are drawn as straight-line segments.
37. **(Unique drawing)** Show that every planar vertex-3-connected graph has a unique planar drawing upto 1) the choice of the outerface, 2) mirror image, and 3) a homeomorphism of the plane.
38. Find the "Kuratowski graphs"(i.e., minimal obstructions) for outerplanar graphs. (A graph is outerplanar if it has a planar non-crossing drawing such that all vertices are incident with the outerface of the drawing.)
39. Show that if $c_v(G) \geq \alpha(G) - 1$ holds true for a graph G with at least 3 vertices, then G has a Hamiltonian path.